

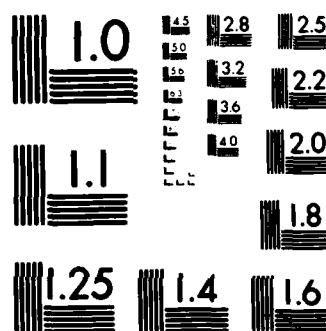
AD-A170 165 ASYMPTOTICALLY ROBUST DETECTION OF STOCHASTIC SIGNALS 1/1
IN CONTAMINATED NOI. (U) TEXAS A AND M UNIV COLLEGE
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SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1. REPORT SECURITY CLASSIFICATION CLASSIFIED		1b. RESTRICTIVE MARKINGS										
2. SECURITY CLASSIFICATION AUTHORITY		3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for Public Release; Distribution Unlimited										
3. CLASSIFICATION/DOWNGRADING SCHEDULE												
4. FORMING ORGANIZATION REPORT NUMBER(S)		5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR- 86-0433										
6. NAME OF PERFORMING ORGANIZATION Texas A&M University	6b. OFFICE SYMBOL (if applicable)	7a. NAME OF MONITORING ORGANIZATION AFOSR/NM										
7c. ADDRESS (City, State and ZIP Code) College Station, TX 77843-3128		7b. ADDRESS (City, State and ZIP Code) Bldg. 410 Bolling AFB, DC 20332-6448										
8a. NAME OF FUNDING/SPONSORING ORGANIZATION AFOSR	8b. OFFICE SYMBOL (if applicable) NM	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR-82-0033										
9c. ADDRESS (City, State and ZIP Code) Bldg. 410 Bolling AFB, DC 20332-6448		10. SOURCE OF FUNDING NOS. <table border="1"><tr><th>PROGRAM ELEMENT NO.</th><th>PROJECT NO.</th><th>TASK NO.</th><th>WORK UNIT NO.</th></tr><tr><td>6.1102F</td><td>2304</td><td>15</td><td></td></tr></table>		PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT NO.	6.1102F	2304	15		
PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT NO.									
6.1102F	2304	15										
11. TITLE (Include Security Classification) Asymptotically Robust Detection of Stochastic Signals in Contaminated Noise		12. PERSONAL AUTHOR(S) M.S. Schnitzer and D.R. Halverson										
13a. TYPE OF REPORT Reprint	13b. TIME COVERED FROM 1/23/85 TO 5/22/86	14. DATE OF REPORT (Yr., Mo., Day) May 22, 1986	15. PAGE COUNT 8									
16. SUPPLEMENTARY NOTATION Proc of the Twenty-Third Annual Allerton Conf. on Communication, Control, and Computing, Oct. 24, 1985.												
17. COSATI CODES <table border="1"><tr><th>FIELD</th><th>GROUP</th><th>SUB. GR.</th></tr><tr><td></td><td></td><td></td></tr><tr><td colspan="3">XXXXXXXXXXXX XXXXXXXXXXXX</td></tr></table>		FIELD	GROUP	SUB. GR.				XXXXXXXXXXXX XXXXXXXXXXXX			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number) Stochastic signals, robust detection, enhanced asymptotic performance.	
FIELD	GROUP	SUB. GR.										
XXXXXXXXXXXX XXXXXXXXXXXX												
19. ABSTRACT (Continue on reverse if necessary and identify by block number) ~ We consider the discrete time detection of stochastic signals in white noise, where the univariate noise density is known perfectly only on an interval about the origin. We present a method to enhance the asymptotic performance of the detector by exploiting this knowledge, and at the same time preserve robustness properties of the detector to the remaining inexact knowledge of the univariate noise density via a saddlepoint condition. We then provide examples to show that improved performance is indeed obtained.												
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED <input checked="" type="checkbox"/> SAME AS RPT. <input type="checkbox"/> DTIC USERS <input type="checkbox"/>		21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED										
22a. NAME OF RESPONSIBLE INDIVIDUAL Major Brian W. Woodruff		22b. TELEPHONE NUMBER (Include Area Code) (202) 767-5027	22c. OFFICE SYMBOL AFOSR/NM									


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ASYMPTOTICALLY ROBUST DETECTION OF STOCHASTIC
SIGNALS IN CONTAMINATED NOISE

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ABSTRACT

We consider the discrete time detection of stochastic signals in white noise, where the univariate noise density is known perfectly only on an interval about the origin. We present a method to enhance the asymptotic performance of the detector by exploiting this knowledge, and at the same time preserve robustness properties of the detector to the remaining inexact knowledge of the univariate noise density via a saddlepoint condition. We then provide examples to show that improved performance is indeed obtained.

I. INTRODUCTION

Consider the discrete time detection of stochastic signals in independent and identically distributed noise. If the underlying statistical distributions are completely known, then selection of a Neyman-Pearson optimal detector is possible. However, in many cases knowledge of the univariate noise density is imperfect, particularly for extreme values of its argument. In such situations the employment of a detector which is robust to the inexact statistical knowledge may be desirable. Earlier work of Kassam and Thomas [1] and Poor, Mami, and Thomas [2] points toward the introduction of limiting for observations of large magnitude in order to impart robustness into the detection scheme. In particular, [2] considers the

Presented at the Twenty-Third Annual Allerton Conference on Communication, Control, and Computing, October 2-4, 1985; to be published in the Proceedings of the Conference.

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asymptotically robust detection of stochastic signals in imperfectly known noise.

In this paper we consider further applications of detectors possessing the same property of limiting. We note that often more is known about the univariate noise density near the origin than on the tails, and such knowledge has the potential to be exploited for improved performance. In some earlier work [3] we show how to design the robust detector by exploiting this knowledge for improved performance over the detector of [1]; this work is applicable to the case of a time varying deterministic signal embedded in imperfectly known noise. We now consider the analogous extension of [3] for improved performance over the detector of [2]. Our results illustrate how such improved performance can be obtained and still preserve the robustness of the detector to the remaining inexact knowledge of the univariate noise density through the employment of limiting.

II. DEVELOPMENT

Suppose we denote the i.i.d. discrete time noise process by $\{N_i; i=1, 2, \dots\}$, with $f(\cdot)$ denoting the associated univariate noise density, and let the stochastic signal be denoted by $\{S_i; i=1, 2, \dots\}$; the zero mean independent signal process is assumed to be independent of the noise. The detection problem then reduces to a choice between

$$H_0 : Y_i = N_i ; i = 1, 2, \dots, n$$

$$H_1 : Y_i = N_i + \theta S_i ; i = 1, 2, \dots, n$$

where we observe realizations y_i of the random variables Y_i . As in [1-3], we assume the noise density $f(\cdot)$ is a member of the Huber-Tukey mixture class, i.e. $f(\cdot)$ can be expressed as $f(\cdot) = (1-\varepsilon)f_0(\cdot) + \varepsilon h(\cdot)$, where ε is a fixed known number between zero and unity, $f_0(\cdot)$ is a known symmetric nominal density, and $h(\cdot)$ is an unknown "contaminating" density. In addition, however, we assume that $f(x)$ is known exactly at each x on an

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interval $[-a, a]$, i.e. $h(x) = f_0(x)$ for all x in $[-a, a]$. We note in passing that a similar assumption is employed in [2] without being directly exploited. Motivated by the results of [1-3], as well as the form of the locally optimal detector given by [2], we employ the class of detectors where a test statistic of form

$T_g(y) = \sum_{i=1}^n E\{S_i^2\} g(y_i)$ is compared to a threshold, with the nonlinearity

$$g(\cdot) \text{ given by } g(x) = \begin{cases} f_0''(x)/f_0(x) & \text{if } |x| \leq k \\ f_0''(k)/f_0(k) & \text{otherwise} \end{cases}$$

We thus see that specifying k effectively specifies the detector.

In order to analyze and compare the performance of the detector, and hence determine k for best performance, an asymptotic fidelity criterion is employed. Because of its relationship to the Pitman-Noether theorem [4], the criterion of Asymptotic Relative Efficiency (ARE) is particularly useful; the associated efficacy functional η provides a convenient measure of performance, where η is defined by

$$\eta(k, h) \triangleq c \left[\int_{-\infty}^{\infty} g''(x) f(x) dx \right]^2, \text{ with}$$

$$\int_{-\infty}^{\infty} g^2(x) f(x) dx$$

$$c = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n \left[E \left\{ S_i^2 \right\} \right]^2. \text{ As}$$

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in other work which employs the ARE, the results of this paper are valid only when various mild regularity conditions are imposed on the underlying statistical distributions. These assumptions are imposed to guarantee that our expressions exist and, when necessary, are nonzero; additionally,



regularity assumptions are required to insure the relevancy of the efficacy functional to the ARE via satisfaction of the hypothesis of the Pitman-Noether theorem [4]. In this paper, our results are presented under the tacit assumption of these numerous though obvious mild conditions. We refer the interested reader to [5-7], in which analogous assumptions are explicitly listed. It should be noted that we do not impose symmetry assumptions on the contaminating density $h(\cdot)$.

To preserve the robustness property of the detector, k is then chosen according to $\sup_k \inf_h \eta(k, h)$, where dependence of η on k and $h(\cdot)$ is indicated. Such a choice of k will be called an optimal choice. The following result provides insight into how to obtain an optimal choice of k :

Theorem: Let

$K = \{k > 0: \text{ if } k > a \text{ then } |g(\cdot)| \text{ and } |g'(\cdot)| \text{ are weakly monotone} \uparrow \text{on } [a, k]\}$. Then a necessary and sufficient condition for k to be an optimal choice over K is that k yield the supremum among

$$(a) \quad \inf_h \eta(k, h) = \frac{2c \left[\int_0^a g''(x) f_0(x) dx + (1-\varepsilon) \int_a^k g''(x) f_0(x) dx \right]^2}{\left[\int_0^a (g^2(x) - g^2(k)) f_0(x) dx + (1-\varepsilon) \int_a^k (g^2(x) - g^2(k)) f_0(x) dx + \frac{1}{2} g^2(k) \right]}$$

if $k > a$

$$(b) \quad \inf_h \eta(k, h) = \frac{2c \left[\int_0^k g''(x) f_0(x) dx \right]^2}{\int_0^k (g^2(x) - g^2(k)) f_0(x) dx + \frac{1}{2} g^2(k)}$$

if $k \leq a$.

Proof: Consider first the proof for part (a). Note that by negative scaling when necessary we can assume without loss of

generality that $g'(\cdot)$ is weakly increasing on $[a, k]$, i.e. $g''(x) \geq 0$ on $[a, k]$. Moreover, by symmetry we also have $g''(x) \geq 0$ on $[-k, -a]$. In a manner similar to the proof of the Theorem of [3] we obtain by direct evaluation

$$\eta(k, h) =$$

$$c \left[2 \left[\int_0^a g''(x) f_0(x) dx + (1-\varepsilon) \int_a^k g''(x) f_0(x) dx \right] + \varepsilon \left[\int_a^k g''(x) h(x) dx + \int_{-k}^{-a} g''(x) h(x) dx \right] \right]^2$$

$$\left[2 \int_0^a (g^2(x) - g^2(k)) f_0(x) dx + 2(1-\varepsilon) \int_a^k (g^2(x) - g^2(k)) f_0(x) dx \right. \\ \left. + g^2(k) + \varepsilon \left[\int_a^k (g^2(x) - g^2(k)) h(x) dx + \int_{-k}^{-a} (g^2(x) - g^2(k)) h(x) dx \right] \right]$$

Noting the weak monotonicity of $|g(\cdot)|$ implies that $g^2(\cdot)$ is weakly increasing, we observe that the numerator of the above is minimized while the denominator is simultaneously maximized if $h(x)=0$ on $[-k, -a] \cup [a, k]$. The expression given for $\inf_h \eta(k, h)$ in part (a) then follows directly.

For part (b) a similar evaluation when $k \leq a$ yields

$$\eta(k, h) = \frac{c \left[2 \int_0^k g''(x) f_0(x) dx \right]^2}{\left[2 \int_0^k g^2(x) f_0(x) dx + 2(1-\varepsilon) g^2(k) \int_k^\infty f_0(x) dx \right. \\ \left. + \varepsilon g^2(k) \left[\int_k^\infty h(x) dx + \int_{-\infty}^{-k} h(x) dx \right] \right]}$$

Noting that $\int_{-k}^{\infty} h(x)dx + \int_{-\infty}^{-k} h(x)dx = 1 - 2 \int_0^k f_0(x)dx,$

we obtain an expression for $\eta(k, h)$ which is independent of $h(\cdot)$ and which agrees with the expression given for $\inf_h \eta(k, h)$ in part (b). QED

We remark that differentiation of the expressions of the Theorem could be employed as in [3] for the purpose of obtaining a condition on an optimal choice k as a solution of a nonlinear equation. However, we have found that for this stochastic signal situation the appropriate equations are typically so complicated that it is no more difficult to search directly over the $\inf_h \eta(k, h)$ values. We also remark that the techniques of [3] could be employed to extend the Theorem to the case of an asymmetric nominal density; the chief cost for such an extension would be the considerably more cumbersome expressions for the $\inf_h \eta(k, h)$ quantity.

We now consider an example which employs a zero mean unit variance sech noise nominal density, i.e.
 $f_0(x) = \frac{1}{2} \operatorname{sech}(\frac{1}{2}\pi x)$ for all x .

We compare the "worst case" efficacy $\eta_{2,w}$ of the detector of [2] (i.e. the minimal efficacy over those $h(\cdot)$ satisfying our constraints as well as those of [2]) with the "worst case" efficacy $\eta_{1,w}$ of our detector with k chosen via the Theorem. The results, which depend on the choice of a and ϵ , are tabulated below:

Table: Values of $\gamma_{1,w}/\gamma_{2,w}$

	$\epsilon = 0.1$	$\epsilon = 0.3$	$\epsilon = 0.5$	$\epsilon = 0.7$
$a=2.00$	6.02	2.65	1.44	1.13
$a=1.50$	6.02	2.65	1.44	1.13
$a=1.00$	3.45	2.73	1.44	1.13
$a=0.50$	2.73	1.55	1.15	1.04

Note that the most dramatic improvement in performance occurs when the greatest certainty is present concerning the univariate noise density (small ϵ and large a). This is probably consistent with what we might expect, however it is also directly opposite to what was observed for the Gaussian example of [3] for the time varying deterministic signal case. Other such opposing effects are known to exist when comparing the stochastic and deterministic signal cases, e.g. respective symmetry and skew symmetry of the nonlinearity $g(\cdot)$, and thus the results illustrated here need not be viewed as surprising when compared to those of [3].

III. CONCLUSION

We have introduced an approach toward improving the asymptotic performance of a detector of a stochastic signal by exploiting available knowledge of the univariate noise density near the origin, while at the same time retaining the robustness of the detector to inexact knowledge of the tails of the noise density. We have shown by way of example that such improved "worst case" performance does indeed occur.

ACKNOWLEDGEMENT

This research was supported by the Air Force Office of Scientific Research under Grant AFOSR-82-0033.

REFERENCES

1. S.A. Kassam and J.B. Thomas, "Asymptotically robust detection of a known signal in contaminated non-Gaussian noise," IEEE Trans. Inform. Theory, vol. IT-22, pp. 22-26, Jan. 1976.
2. H.V. Poor, M. Mami, and J.B. Thomas, "On robust detection of discrete time stochastic signals," J. Franklin Inst., vol. 309, pp. 29-53, Jan. 1980.
3. D.R. Halverson, "Asymptotically robust detection in partially contaminated noise," Circuits, Systems, and Signal Processing, vol. 3, pp. 493-503, 1984.
4. G.E. Noether, "On a theorem of Pitman," Ann. Math. Stat., vol. 26, pp. 64-68, 1955.
5. H.V. Poor and J.B. Thomas, "Memoryless discrete-time detection of a constant signal in m -dependent noise," IEEE Trans. Inform. Theory, vol. IT-25, pp. 54-61, Jan. 1979.
6. D.R. Halverson and G. L. Wise, "Discrete-time detection in ϕ -mixing noise", IEEE Trans. Inform. Theory, vol. IT-26, pp. 189-198, March 1980.
7. D.R. Halverson and G. L. Wise, "Asymptotic memoryless detection of random signals in dependent noise", J. Franklin Inst., vol. 312, pp. 13-29, July 1981.

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